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Proof schemes used by first grade of preservice mathematics teachers about function topic

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Abstract

The purpose of this study is to investigate how the preservice teachers justify their solutions for mathematics problems. With this purpose an examination was applied to 40 preservice teachers which were at the first grade of the department of elementary mathematics teacher education. Then 6 preservice teachers were randomly selected from these preservice teachers and clinical interviews were conducted with each of them. During the interviews 10 problems were asked to each preservice teachers. According to the data results, it's seen that preservice teachers solve the problems with different ways. The analysis of the data showed that preservice teachers usually used one of the empirical schemes, which was the example based in their justification.

Keywords: *Proof schemes, proof, mathematics education, preservice teachers;*

1. Introduction

Proofs are most important of something that making mathematics to mathematics (Padula, 2006). Because, proofs supply correctness and incorrectness of every situations in mathematics (Tall & Mejia-Ramos, 2006). Not only proofs show whether a situation is correct or wrong, but also proofs show why it is wrong or correct (Hanna, 2000). Proving is defined as a mental action used for removing doubts about any claim mentioned by a society or an individual (Harel ve Sowder, 1998; Harel ve Sowder, 2007; Harel, 2008b). Because of this, proof is very important at mathematics.

According to Bell (1976), mathematical proof is defence, explain and systematization. In addition one of the most important function of proofs is to prepare confidence for students. Second function of proofs for students is understanding one result and why this result is correct. Third function is explaining logical concept of ideas, brainstorming and deduction (Bell, 1976). According as these three functions, proof is completed at three stages. First stage is confirmation; second stage is explanation and third stage is abstraction (Baki, 2009).

Many teachers at classroom environment don't arrange proofs thinking their values (Knuth, 1999; Knuth, 2002). At traditional classrooms teachers show proofs without giving chance to students for composing proofs. At this situation students do not active at constructing knowledge and they are just passive knowledge obtainer (Harel & Sowder, 1998). However proofs give opportunity to students for realizing mathematical facts themselves without

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trusting teachers and books (Knuth, 1999; Knuth, 2002). Also proofs have importance place for developing and changing student's mathematical thinking.

Proof schemes at proving process offer mental stages of students' mathematical development. Proof schemes are categorized three main schemes which are external, empirical and analytical and sub schemes of them (Sowder & Harel, 1998). Students predicate what they learned at mathematics using external schemes to books and other people. In addition they accept correctness of an idea when it is said by trustful person (Flores, 2002; Flores, 2006). External schemes are separated three sub schemes; authoritarian, ritual and symbolic. Empirical schemes consists special samples and intuitive figures which are used for reliability students. Empirical schemes are separated two sub categories which are perceptual and example-based (Harel & Sowder, 1998). Analytical schemes supply logical inference reliability for estimation. Analytical proof schemes are separated two part: transformational and axiomatic.

Proof schemes are important for observing thinking reactions of students at mathematical situations. Because, proof schemes show a student's ideas and choice about correctness and incorrectness of one mathematical situation (Harel & Sowder, 1998). Also proof schemes help to understand learning level of students (Knapp, 2006). Although proof schemes have a lot of advantages and useful roles, students at different level have problems to understand, like and compose proofs (Martin & Harel, 1989; Moore, 1994; Harel & Sowder, 1998; Jones, 2000; Güven et al., 2005).

Graduate students and preservice mathematic teachers have problems to compose logic ,making proof and working with proof strategies (Senk, 1983; Moore, 1994; Harel & Sowder, 1998; Dreyfus, 1999; Almeida, 2000; Jones, 2000; Recio & Godino, 2001; Weber, 2001; Weber, 2004; Stylianides et al., 2005; Stylianides et al., 2007). This affects their profession life when they become a teacher (Weber, 2001; Moralı et al., 2004). Because, beliefs and ideas of teachers affect their own behaviors (Erickson, 1993). For this reason, to plan and implement mathematics lessons effectively teachers have to know concepts, and mathematical ideas and where from these concepts and ideas arisen (Moralı et al., 2006). Because, teachers' perceptions and attitudes about proofs affect development of student's proof skills and experiences (Almeida, 2003). After proofs, proof schemes are used.

Proof schemes showing students' thinking reactions (Harel & Sowder, 1998; Sowder & Harel, 1998; Flores, 2002; Flores, 2006) but in Turkey, there are just few research about proof schemes. Because of this aim of this study is investigating proof schemes used by first grade preservice elementary mathematics education teachers and basic problem of this study is "What are the proof schemes that are used by first grade of preservice elementary mathematics education teachers at function subject?"

2. Method

At this study, written exams were used for collecting quantitative data and clinical interviews were used to collect qualitative data. At this study, aiming to focus on proof schemes that are used by preservice mathematics teachers, qualitative data were emphasized. Quantitative data were collected from 40 first grade preservice mathematics teachers and qualitative data were collected from 6 preservice teachers which selected randomly.

10 open-ended problems were prepared for revealing proof schemes at function subject which used by preservice elementary mathematics education teachers. For this applied preservice teachers both written exam and clinical interviews. Clinical interviews were made one month later from written exam.

The data were analyzed with using descriptive analyze method after the cassette transcription of the interviews. During the analyzing process, the justifications of the students' were mostly emphasized. For each problem, the researcher and a field expert determined which proof scheme was used by each student. At the interview analyze, to determine the proof schemes of students, their last expressions of the problem solution were utilized. Findings indicate that proof schemes used by students were consistent with Sowder and Harel's (1998) research.

3. Findings

Results of study showed that first grade preservice elementary mathematics education teachers used three proof schemes at written exams. Written exam having 10 questions were applied to 40 preservice teachers. According to results, empirical schemes (%28,25, 113 problem) were used most. External (%24,25; 97 problem) and analytical

schemes (%24,75, 99 problem) were used at same level. Also preservice teachers (91 problem) did not solve %22,75 of problems

After written exam clinical interviews were made with six preservice teachers. At this process, preservice teachers solved most of the problems but some of them can't be solved by them. External, empirical and analytical schemes were used for clinical interviews and they are varied according to types of problem types (look Table 1.). At this process, preservice teachers were used mainly empirical proof schemes with example-based. Types of external schemes were generally authoritarian proof schemes and analytical proof schemes were generally axiomatic proof schemes. In addition preservice teachers mostly used external schemes at first problem and empirical schemes were used at second problem. Analytical schemes were used mostly at tenth problem and some problems could not be solved by preservice teachers. Evaluating the problems at clinical interviews all proofs were evaluating not considering whether solutions are correct or wrong and all proofs were classified.

Table 1. Proof schemes used by preservice teachers at clinical interviews

	External	Empirical	Analytical	Blank	Total
Problem1	5	-	1	-	6
Problem2	1	5	-	-	6
Problem3	-	3	3	-	6
Problem4	2	2	1	1	6
Problem5	1	3	1	1	6
Problem6	-	3	3	-	6
Problem7	1	3	-	2	6
Problem8	2	-	4	-	6
Problem9	1	3	2	-	6
Problem10	-	-	6	-	6
Total	13	22	21	4	60

There are not big differences at used proof schemes used by preservice mathematics teachers at the process of correcting problems. When we investigate results three of preservice mathematic teachers (Ebru,Filiz and Zekiye) did not use empirical schemes any time and Ebru whom with clinical interview made, did not use any empirical proof schemes at interviews (look Table 2.). But other participants used three proof schemes both written exams and interviews.

Table 2. The proof schemes which the preservice teachers used according to the examination and clinical interviews

	Ebru		Ahmet		Filiz		Hande		Lütfi		Zekiye	
	E	**CI**	**E**	**CI**	**E**	**CI**	**E**	**CI**	**E**	**CI**	**E**	**CI**
P1	EX	EX	EX	EX	DI	A	EX	DI	EX	DI	EX	DI
P2	EX	EX	EM	EM	B	EM	EX	EM	EM	EM	B	EM
P3	A	A	EM	EM	A	A	EM	EM	A	EM	A	A
P4	B	EX	EM	EM	A	DI	A	A	B	B	B	EM
P5	EX	EX	EM	EM	A	A	EX	EM	EM	EM	B	B
P6	B	A	EM	EM	A	A	EM	EM	EM	EM	A	A
P7	EX	EX	EM	EM	B	B	EM	EM	EM	EM	B	B
P8	A	EX	A	A	A	A	A	A	A	DI	A	A
P9	EM	A	EM	EM	EX	DI	EM	EM	EM	EM	A	A
P10	A	A	A	A	A	A	A	A	A	A	A	A

E: Examination, CI: Clinical Interview, EX: External, EM: Empirical, A: Analytical, B: Blank

The examples of the proof schemes used by the preservice teachers in the interviews will be presented in the following chapters and in this process, the participants in the study names are given by the researcher.

3.1. External proof schemes

Participant didn't make proof when used external proof schemes without using any source (book, teacher, parent) and also participant used old knowledge and unmeaning symbols. Preservice teachers used 13 times external schemes at 60 problems in the interviews. Authoritarian proof schemes were used mainly at external schemes.

At this process Ebru used mainly external schemes at written exams. Participant solving first problem used external schemes both written exams and clinical interviews. Ebru made explanation below and she used authoritarian proof schemes solving at the first problem. She firstly gave definition of function. And later she continued her explain;

T : Is your definition enough for making one expression a function?

Ebru: May be exactly no., But I think it is enough

T :Where from did you remember this estimation?

Ebru: I think it is. We have to find expression when we define ideas at description group.

T : Why do you think at this way?

Ebru: We learned like that.

T : Where from did you learn?

Ebru: From my old experiences and I learned it at mathematics lessons like it. Also some extra things were mentioned at classroom but that is all that I remember.

Preservice teacher explained expression that why expression is not a function with her knowledge that she learned at classroom. But this preservice teacher remember this expression wrongly that she have seen before at mathematics classroom. Most important reason of this can be memorazing expression instead of mentally composed by preservice teachers.

3.2. Empirical proof schemes

Preservice teachers made their proofs using their previous experiences and perception and that used sample drawings making empirical proof schemes. When we look to results of interviews 22 preservice teachers were used empirical proof schemes. Preservice teachers were used mainly example-based schemes of empirical proof schemes.

Ahmet and Lütfi defined one f and g function to show correctness of $(fog)^{-1}=g^{-1}of^{1}$ expression at seventh problem. Later they have found resultant function as next step they have composed inverse function. At the end of the two process they have obtained same result and they told that expression is correct.

Hande used equally to proof schemes at written exam. This participant used empirical schemes at written exam and interviews. Hande is one of the participant choosing to define function solving to problem. She solves expression and defines ;

"f as unit function f(x) is equal x ,sorry at this point I(x) is equal x. Let a f(x) function be ax+b and it is any function. Later we will find resultant function putting I to f, f(x)=ax+b. At I, I(x)is x ,and this is f(x). f(x)=ax+b. Now f(x) function is ax+b.Because of I(x) is unit function, I(x) is also ax+b. They are equal each othe . That is all."

She made that confirmation and she preferred to write ax+b instead of f function. Participant explained question of "Why did you choose f function as ax+b" as it is an normal function. ax+b function is a normal function for participant and this function is enough for participant to correct equality. Participant used ax+b to define 3x+5 as unit function at this process.

Preservice teachers using perceptual schemes at empirical proof schemes defined process as "it must be done with this way, it is correct when it is made like this, it is certain that it is correct, I feel it is correct.

3.3. Analytical proof schemes

Participants used definitions, terminologies, reason/result relation and designs at analytical proof schemes. In the interviews when preservice mathematics teachers solving problems used axiomatic proof schemes for 21 problems.

Filiz is a participant who use mainly analytical schemes at written exam. Filiz used analytical schema solving fifth problem at written exam and interviews. She explained at interview;

"According to what I associated it as covering? I could not solve it. f(A) is covering, A is covering and f function is covering; f(A) is equal B. For example display cluster is equal to value cluster, it is covering. Now from this point, display cluster f(A) is equal B. Second situation here is if f(A) is equal C at g(x), we will say it is covering. Now it is

okay. We have found f(A) and we know g(B) is C. We say value cluster is covering because it is equal to display cluster."

She used axiomatic proof schemes. Preservice teacher reached definition of covering with mental operations and she explained g and f functions are covering. She also transferred to g resultant function to f function.

Zekiye used analytical schemes showing to $(f^{-1})^{-1}$=f expressions and she explained it;

"It is inverse of f's inverse, and it is equal y. This is from rule. Using rule inverse of y is equal x. We have used inverse again. Double inverse of f(x) =y is equal y Both of them are equal each other."

Participant used previously learned rules with analytical schemes to show correctness of her expressions.

4. Discussion, Conclusions and Suggestions

At this study, 10 problems were offered to 6 students with written exams and interviews about functions subject. Most of them are solved by preservice teachers, but some of them could not be solved by them. External, empirical and analytical schemes were used at written exams and interviews. Used external schemes appeared mainly as authoritarian proof schemes and analytical schemes were appeared as axiomatic proof schemes. At this process, preservice teachers used example-based of empirical schemes at written exams and interviews. At similar studies made with preservice teachers mainly empirical proof schemes were used (Sowder & Harel, 1998; İskenderoğlu, 2010). One of the reason of this can be; preservice teacher find a dew sample correcting to given situation at problem and they see this enough to accept. Another reason is preservice teachers see this method simple and easy. Also they see concrete solutions to trust process.

Although preservice teachers do not vary in terms of used proof schemes, some of them did not use empirical schemes any time. One of the reasons of this is effects of their old experiences and differences to their approach to proofs. Because one of the reason effecting their approach to proof is their own views (Healy & Hoyles, 2000).

Participants solved most of the problems showing explanations. But at written examples nearly %25 percent of problems did not solved by participants. One of the reasons of this can be time limitation. At interviews there was no time limitation and preservice teachers have enough time to solve problems. In addition some preservice teachers could not explain their solutions although they have made correct or wrong. Reason of this situation they were not sure about their solutions. Also they can't have enough information to solve problems.

For future research, relation between proof schemes used by elementary preservice mathematics teacher and views of them about proof schemes can be investigated. And also question of "Why preservice mathematics teacher use empirical schemes commonly" can be researched.

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